

Generating f_{NL} at $\ell \lesssim 60$

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Abstract

The CMB anisotropy at $\ell \lesssim 60$ seems to have some special features which include (i) a dipole modulation and (ii) a decrease in power. It is known that both of these effects can be generated if a curvaton-type field has a super-horizon perturbation. It is also known that this will generate non-gaussianity f_{NL} in the same range of ℓ , whose magnitude has a lower bound coming from the magnitude of the observed CMB quadrupole. I revisit that bound in the present paper, and point out that it may or may not be compatible with current data which should therefore be re-analysed.

1 Introduction

For multipoles $\ell \lesssim 60$ (corresponding to large scales $k^{-1} = x_{\text{ls}}/\ell \gtrsim 200$ Mpc where $x_{\text{ls}} = 14,000$ Mpc is the distance of the last scattering surface) the CMB anisotropy seems to have some special features. One of these is a dipole asymmetry [1, 2]

$$\Delta T = (1 + A(\hat{\mathbf{p}} \cdot \hat{\mathbf{n}})) \Delta T_{\text{iso}}(\hat{\mathbf{n}}), \quad (1)$$

with $A = 0.07 \pm 0.02$. The effect seems to be a real one [3], despite an earlier suggestion to the contrary [4]. It must be scale-dependent though, because observation requires [5, 6] $A < 0.0045$ (95% confidence level) for $\ell = 601\text{--}2048$ corresponding to $k^{-1} \sim 10$ Mpc. Another feature [1, 2, 7], shown in Figure 1, is a suppression in the magnitude of the multipoles, below the level that would correspond to the Λ CDM model with constant spectral index $n_s = 0.96$ that provides a good fit at higher ℓ .

Making the usual assumption that the CMB anisotropy is generated by the primordial curvature perturbation ζ , these features correspond to

$$\zeta_{\mathbf{k}}(\mathbf{x}) = (1 - C_\zeta(k) + A_\zeta(k)\hat{\mathbf{p}} \cdot \mathbf{x}/x_{\text{ls}}) \zeta_{\mathbf{k}}, \quad (2)$$

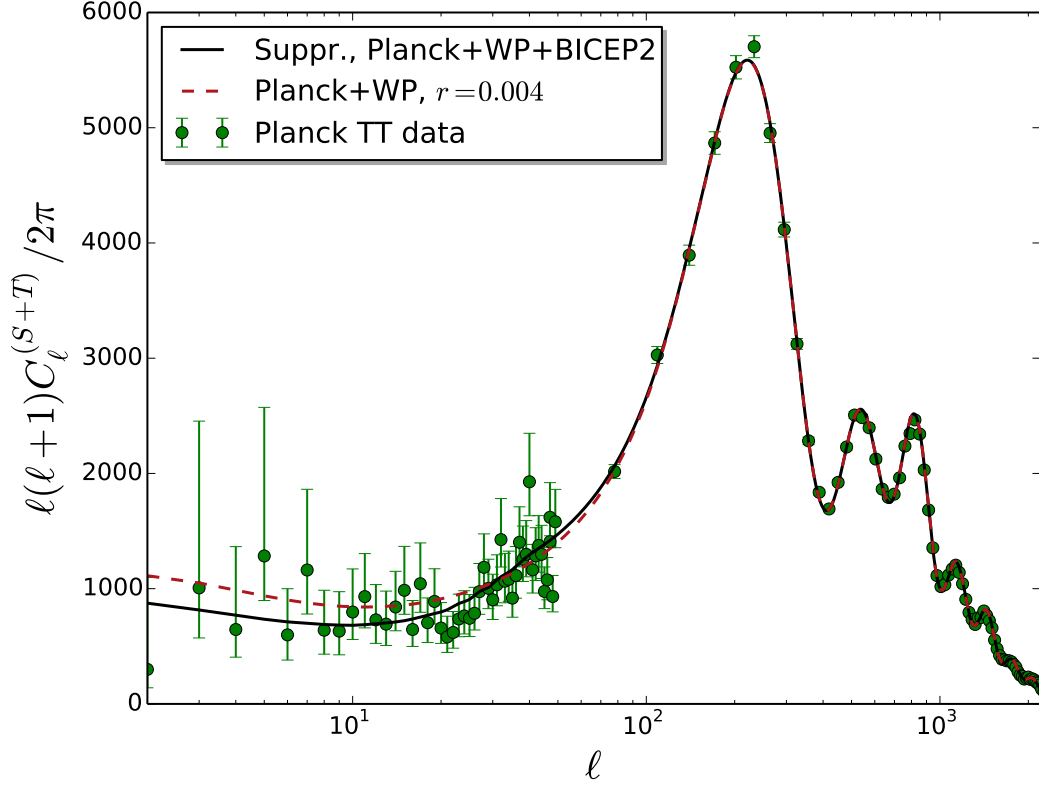


Figure 1: The CMB multipoles as shown in [8]. The upper line is the Λ CDM model best fit with constant $n_s = 0.96$. The lower gives the best fit of [8].

and

$$\mathcal{P}_\zeta(k, \mathbf{x}) \simeq (1 - 2C_\zeta(k) + 2A_\zeta(k)\hat{\mathbf{p}} \cdot \mathbf{x}/x_{\text{ls}}) \mathcal{P}_\zeta(k). \quad (3)$$

This expression, and its meaning was given in [10]. The quantity $\zeta_{\mathbf{k}}$ is a statistically homogeneous quantity, which fits the higher multipoles and has the constant spectral index $n_s = 0.96$ that is required by observation. The quantity $\zeta_{\mathbf{k}}(\mathbf{x})$ is also statistically homogeneous, but it is defined only within a box at the position \mathbf{x} , whose size is much smaller than x_{ls} . For that to make sense, I consider only $k^{-1} \gg x_{\text{ls}}$. To account for the dipole asymmetry we need $A_\zeta(k) \sim 0.07$ for $k^{-1} \gtrsim x_{\text{ls}}/60$, but the distribution of distant quasars [9] requires $A_\zeta < 0.014$ (99% posterior probability) on the scale $k^{-1} \sim 1$ Mpc.

The form Eq. (3) for ζ can arise if it is generated by a curvaton-type field χ (one not affecting inflation) whose contribution has a significant contribution from super-horizon scales [10, 11, 12]. Then we have [10]

$$\frac{6}{5}f_{\text{NL}}(k)\zeta_{\text{L}}(\mathbf{x}) = -C_\zeta(k) + A_\zeta(k)\hat{\mathbf{p}} \cdot \mathbf{x}/x_{\text{ls}} + \dots, \quad (4)$$

where $f_{\text{NL}}(k)$ is the reduced bispectrum in the equilateral configuration and ζ_{L} is the super-horizon contribution to ζ evaluated to linear order in $\delta\chi$. To make f_{NL} scale-dependent we need non-linear evolution of χ during inflation [13], corresponding to a potential which isn't quadratic.

The perturbation ζ_L generates contributions to the CMB quadrupole, which (barring a strong cancellation) must be \lesssim the observed quantity. This places a lower bound on the magnitude of $f_{\text{NL}}(k)$. In the present paper I revisit the bound, and its significance.

2 Contributions to ζ

I first discuss the various contributions to ζ , clearing up some minor discrepancies between [11, 10, 12, 14], and a major discrepancy with the formula given in [15, 16, 17].

If ζ is generated from the perturbation of a curvaton-type field χ , the non-linear δN formula gives [20, 21]

$$\zeta(\mathbf{x}) = N(\chi(\mathbf{x})) - N(\chi_0) \quad (5)$$

$$= N'(\chi_0) (\delta\chi_S(\mathbf{x}) + \delta\chi_L(\mathbf{x})) + \frac{1}{2} N''(\chi_0) (\delta\chi_S(\mathbf{x}) + \delta\chi_L(\mathbf{x}))^2 + \dots \quad (6)$$

$$\equiv (\zeta_S(\mathbf{x}) + \zeta_L(\mathbf{x})) + \frac{3}{5} f_{\text{NL}}(k) (\zeta_S(\mathbf{x}) + \zeta_L(\mathbf{x}))^2 + \dots \quad (7)$$

$$= \zeta_S(\mathbf{x}) + \frac{3}{5} f_{\text{NL}}(k) \zeta_S^2(\mathbf{x}) + \frac{6}{5} f_{\text{NL}}(k) \zeta_L(\mathbf{x}) \zeta_S(\mathbf{x}) \\ + \zeta_L(\mathbf{x}) + \frac{3}{5} f_{\text{NL}}(k) \zeta_L^2(\mathbf{x}) + \dots, \quad (8)$$

where $\zeta_S \equiv N'(\chi_0) \delta\chi_S(\mathbf{x})$ and $\zeta_L \equiv N'(\chi_0) \delta\chi_L(\mathbf{x})$. I have equated N''/N'^2 with $(6/5)f_{\text{NL}}(k)$, where $f_{\text{NL}}(k)$ is the reduced bispectrum of ζ in the equilateral configuration, which will be justified shortly.

To explain the dipole anisotropy, this expression is taken to apply within a comoving box a bit bigger than x_{ls} . The left hand side is time-independent, but every object on the right hand side depends on time, which is chosen to be the epoch t_k of horizon exit $k = a(t_k)H(t_k)$ for a scale $k \gg x_{\text{ls}}^{-1}$. I am using k instead of t_k to denote the epoch. The unperturbed quantity χ_0 is the average within the box. The perturbations $\delta\chi_S$ and $\delta\chi_L$ are evaluated on the flat slicing. The perturbation $\delta\chi_S$ is generated by the vacuum fluctuation on scales $1/k' \leq 1/k$, and the perturbation $\delta\chi_L$ comes from scales $1/k' \gg x_{\text{ls}}$ (super-horizon scales).

The first two terms of Eq. (8) give $\zeta_{\mathbf{k}}$. The dominant first term gives the spectrum $\mathcal{P}_\zeta(k) = N'^2(\chi_0(k))(H(k)/2\pi)^2$. Since $\mathcal{P}_\zeta(k)$ and $H(k)$ are both slowly varying, so is N' . Including the second term, one can verify [13] that $f_{\text{NL}}(k)$ is indeed the reduced bispectrum in the equilateral configuration.¹

Omitting the small middle term, the first line of Eq. (8) gives $\zeta(\mathbf{x}, k)$, which means that

$$\frac{6}{5} f_{\text{NL}}(k) \zeta_L(\mathbf{x}) = -C_\zeta(k) + A_\zeta(k) \hat{\mathbf{p}} \cdot \mathbf{x} / x_{\text{ls}} + \dots, \quad (9)$$

¹This is Eq. (3.5) of [13], evaluated in the equilateral configuration with t_i the epoch of horizon exit and with the second term set to zero. The justification for ignoring the second term is given, with references, in the paragraph following the one containing Eq. (3.5).

where the dots indicate sub-dominant quadratic and higher terms. I have ignored the dependence of ζ_L upon t_k . This is justified because ζ_L must be very slowly varying, since it gives the dominant contribution to ζ on super-horizon scales which itself is independent of t_k . We see that C_ζ and A_ζ are both proportional to f_{NL} .

In [15, 16, 17] they give a formula (Eq. (15) of [15]) which seems to be equivalent to Eq. (9), with the crucial difference that $f_{\text{NL}}(k00)$ there denotes the squeezed configuration. But the formula is not really equivalent, because their quantity $\mathcal{R}(t, \mathbf{x})$, defined in [15], is not the same as ζ . That is because their $\mathcal{R}(t, \mathbf{x})$ is defined in Eq. (2) which refers to the slicing of uniform *curvaton* field perturbation. In contrast, $\zeta(t, \mathbf{x})$ is defined by that formula, but referring to the slicing of uniform energy density. During inflation, the curvaton field is supposed to have a negligible effect which means that the slicing of uniform energy density is the same as the slicing of uniform *inflaton* field [19]. Even if we identify their $\mathcal{R}(\mathbf{x})$ with our $\zeta(\mathbf{x})$ (where now the argument t is omitted because we deal with the late-time quantity that is constrained by observation) Eq. (9) of [15] is not valid. This equation is valid for ζ during inflation, as discussed in [18]. It is not valid for the observed ζ when that is generated by the perturbation of a curvaton-type field. As they recognise, their Eq. (9) is the key to the rest of their analysis, which is therefore invalid.

In this paper I find that the effect being considered generates the equilateral configuration, not the squeezed configuration. That is despite the fact that the effect comes from a field perturbation on a super-horizon scale.

Although it is possible for a curvaton-type model to give the desired result Eq. (9), one should be clear that it is not easy. Indeed, when the rate of change of f_{NL} is small it is given by [22]

$$\left| \frac{df_{\text{NL}}(k)}{d \ln k} \right| = \frac{5}{6} \sqrt{\frac{r}{8}} \frac{M_{\text{P}} |V'''}{3H^2}, \quad (10)$$

where $r \simeq 0.16$ is the tensor fraction, $H \simeq 1 \times 10^{14}$ GeV is the Hubble parameter during inflation, $M_{\text{P}} = 2 \times 10^{18}$ GeV is the reduced Planck mass and $V(\chi)$ is the potential of the curvaton-type field. We need $|V'''|$ to small enough to keep the rate of change small while ζ is being generated on scales $k^{-1} \lesssim x_{\text{ls}}/60$, but we need $|V'''|$ to be much bigger before that. This difficulty is of course merely a reflection of the fact that we are talking about features that is present only on scales $k^{-1} \gtrsim x_{\text{ls}}/60$, and any other explanation of the features would have a similar difficulty.

3 The forms of $A_\zeta(k)$ and $C_\zeta(k)$

In the next section I am going to produce lower bounds on $|f_{\text{NL}}(k)|$ in terms of $A_\zeta(k)$ and $C_\zeta(k)$. To obtain numerical values for the bound I will have to assume something about the latter quantities, which are both proportional to $f_{\text{NL}}(k)$.

Within the large observational uncertainty, the suppression of multipoles can be fitted with a step function [8],

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_0) (1 - K\theta(k_0 - k)), \quad (11)$$

with a best fit corresponding to $K = 0.35$ and $k_0^{-1} < x_{\text{ls}}/38$. This would correspond to $C_\zeta(k) = 0.17\theta(k_0 - k)$. Taken literally, the step function Eq. (11) would correspond to a delta function for V''' in Eq. (10) but a monotonic variation of V''' may also be compatible with the data.

A step function for $C_\zeta(k)$ implies a step function for $A_\zeta(k)$, and I will take $A_\zeta(k) = 0.07\theta(k_0 - k)$. To cover the full range $\ell < 60$ on which the anisotropy exists we need $k_0^{-1} = x_{\text{ls}}/60$, which is a bit smaller than the value in the previous paragraph. In view of the large uncertainties in the data I will ignore the difference, and for definiteness adopt $k_0^{-1} = x_{\text{ls}}/60$.

Instead of the step function forms for A_ζ and C_ζ that I am adopting, [23, 24] takes $A_\zeta \propto C_\zeta \propto k^{-n}$. To get sufficient suppression on scales $k^{-1} < x_{\text{ls}}/60$ one needs $n > 0.56$, but this also causes strong scale dependence on scales $k^{-1} > x_{\text{ls}}/60$. For instance, setting $C_\zeta(k) = 0.15$ at the scale $k^{-1} = x_{\text{ls}}/30$ and adopting the minimal scale dependence $n = 0.56$ gives complete suppression ($C_\zeta = 1$) at the scale $k^{-1} = x_{\text{ls}}$. It is shown in [23, 24] it is shown that the strong k -dependence causes $\mathcal{P}_\zeta(k)$ to be different from the CMB anisotropy parameter $A(\ell)$.

Although the power-law form for $C_\zeta(k)$ is very different from the step function form, the forms of the suppression of the multipoles might not be so different, at least if n is not too far above 0.56. In that case the power law form for $C_\zeta(k)$ could be as viable as the step function. Using it would make little difference to my conclusions, the main effect being the reduction of the large-scale $A_\zeta(k)$ below the value 0.07 that I am adopting. For $n = 0.56$ the reduction according to [24] is by roughly one half.

4 The EKC and GZ effects

The EKC and GZ effects refer to the quadrupole a_{2m} of the CMB anisotropy. The EKC effect [11] comes from the last term of Eq. (8). To bound it one uses the data analysis of [25] which gives

$$\sqrt{\frac{1}{5} \sum |a_{2m}|^2} = 6.5 \times 10^{-6}. \quad (12)$$

Using the Sachs-Wolfe approximation

$$\Delta T(\mathbf{e})/T = (1/5)\zeta(x_{\text{ls}}\mathbf{e}), \quad (13)$$

and requiring that the EKC contribution to the left hand side of Eq. (12) is less than the total, one finds [14]²

$$|f_{\text{NL}}(k)| \gtrsim 66 \left(\frac{A_\zeta(k)}{0.07} \right)^2. \quad (14)$$

The GZ effect [26] is the contribution to the CMB quadrupoles a_{2m} coming from the quadratic part of ζ_L , through the next-to-last term of Eq. (8).³ To estimate the GZ effect, one has to assume that we occupy a typical location, within some box large enough to contain the wavelengths that contribute to $\delta\chi_L$. For simplicity I will assume that there is just one wavenumber k_L and write.

$$\mathcal{P}_{\zeta_L}(k) = P^2 \delta(\ln k - \ln k_L) \quad (15)$$

Equating $\zeta_L^2(0)$ with $\langle \zeta_L^2 \rangle$ we have

$$|C_\zeta(k)| = \frac{6}{5} P |f_{\text{NL}}(k)|. \quad (16)$$

Equating $|\nabla \zeta_L(0)|^2$ with $\langle |\nabla \zeta_L|^2 \rangle$ we find

$$P \frac{k_L}{a_0 H_0} = \left| \frac{A_\zeta(k)}{3.7 f_{\text{NL}}(k)} \right|, \quad (17)$$

where I used $1/H_0 = 4,500 \text{ Mpc}$ instead of x_{ls} with an eye to the open universe calculation. Since $A_\zeta(k)$ is proportional to $f_{\text{NL}}(k)$ the right hand side is independent of k .

Using Eqs. (16) and (17),

$$k_L/a_0 H_0 = 0.23 \left| \frac{A_\zeta(k)}{C_\zeta(k)} \right| \quad (18)$$

We also need the $C_2 \equiv \langle |a_{2m}|^2 \rangle$.⁴ In a flat universe, the GZ contribution to

$$C_2^{\text{GZ}} = 0.21 \left(\frac{k_L}{a_0 H_0} \right)^4 P^2 \quad (19)$$

²The normalization of this result is almost the same as the one given in [11], who integrate an approximate evolution equation. (Both their procedure and the Sachs-Wolfe approximation ignore the primordial anisotropic stress of the neutrinos which gives an error of tens of percent.) Both [11] and [14] take f_{NL} to be a constant. It was pointed out in [10] that the result holds in any curvaton-type model (ie. a model that generates ζ from the perturbation of a field different from the inflaton), allowing $f_{\text{NL}}(k)$. The normalization in [10] is a bit too low though, because it referred to the arXiv v1 of that paper which was later corrected; I thank A. Erickcek for clarifying this.

³Contrary to the statement in [14], the linear part of the next-to-last term of Eq. (8) has no physical effect. This is because the linear contribution to ζ has no physical effect by virtue of the adiabatic initial condition, which is valid in our setup. That is the case for both a flat [27] and an open [28] universe. Contrary to the statement in [10], the EKC effect has nothing to do with the GZ effect.

⁴This is the standard definition of C_2 (see for instance [25]). In [11, 14, 12], C_2 is instead taken to denote the observed quantity $\frac{1}{5} \sum |a_{2m}|^2$.

Let us assume that

$$\sqrt{C_2^{\text{GZ}}} \lesssim \sqrt{\frac{1}{5} \sum |a_{2m}|^2} = 6.5 \times 10^{-6}. \quad (20)$$

Then Eqs. (17), (18), and (19) need⁵

$$|f_{\text{NL}}(k)| \gtrsim 170 \left(\frac{0.17}{|C_\zeta(k)|} \right) \left(\frac{A_\zeta(k)}{0.07} \right)^2. \quad (21)$$

The open universe case has been considered in [12], where they point out that the super-horizon contribution is generated in a previously proposed open universe inflation model [29]. This slow-roll model is incompatible with complete dominance of the curvaton-type contribution, but it could allow a significant curvaton-type contribution which would be enough for our purpose. Let us proceed without tying ourselves to any particular inflation mechanism.

For an open universe, it is convenient to choose $1 - \Omega_0 = 1/(a_0 H_0)^2$. Then curvature scale corresponds to $k = 1$. The previous result applies if $k_L \gg 1$ (sub-curvature scale), but things are different if $k_L \ll 1$ (super-curvature scale). Instead of Eq. (19) we have [30]

$$C_2^{\text{GZ}} = 0.00543 (1 - \Omega_0)^2 k_L^2 P^2, \quad (22)$$

and Eq. (20) gives

$$(1 - \Omega_0) k_L P < 8.8 \times 10^{-5} \quad (23)$$

and Eq. (17) can be written

$$(1 - \Omega_0)^{1/2} k_L P = \left| \frac{A_\zeta}{3.7 f_{\text{NL}}} \right|. \quad (24)$$

After using Eq. (16) to eliminate P , Eqs. (22) and (23) become

$$(1 - \Omega_0) k_L < \frac{6}{5} \frac{|f_{\text{NL}}|}{|C_\zeta|} \times 8.8 \times 10^{-5} \quad (25)$$

$$(1 - \Omega_0)^{1/2} k_L = \frac{6}{5} \left(\frac{|A_\zeta|}{3.7 |C_\zeta|} \right). \quad (26)$$

Eliminating k_L from these gives

$$\frac{5}{6} |f_{\text{NL}}| > \left(\frac{(1 - \Omega_0)^{1/2}}{8.8 \times 10^{-5}} \right) \left(\frac{|A_\zeta|}{3.7} \right) \quad (27)$$

⁵This may be compared with the bound obtained in [10], where only $|C_\zeta| \ll 1$ was assumed. Setting $C_\zeta = 1$ in my expression, it is three times bigger (within a rounding error) than the one found in [10], because the latter paper used a bound on $\sqrt{C_2^{\text{GZ}}}$ that was a factor 3 looser than Eq. (20).

Using $k_L \ll 1$, Eq. (26) becomes

$$\left(\frac{6}{5}\right) \frac{|A_\zeta|}{3.7|C_\zeta|} \frac{1}{(1 - \Omega_0)^{1/2}} \ll 1, \quad (28)$$

leading finally to

$$|f_{\text{NL}}| \gg 48 \left(\frac{0.17}{|C_\zeta(k)|} \right) \left(\frac{A_\zeta(k)}{0.07} \right)^2. \quad (29)$$

This is a bit different from the result in [12] because they don't use Eq. (16) and just require $P < 1$.

The tightest bound is for the flat case, corresponding to the GZ bound Eq. (21). It can be reduced from the fiducial value 170 by a factor 2 if we reduce $A_\zeta(k)$ by $1-\sigma$, or if we take on board the reduction of $A_\zeta(k)$ found in [24]. It can be further reduced by say a factor three if we multiply the right hand side of Eq. (20) by a factor three as was done in [10]. (Such a factor allows for some degree of cancellation between the GZ effect and the ordinary contribution to the quadrupole, and for the fact that we might not occupy a typical position within the large box.) That would give $|f_{\text{NL}}| < 30$.

Current observational bounds on f_{NL} take it to be independent of the overall scale. Depending on the shape that is adopted, one finds [31] $|f_{\text{NL}}| \lesssim 10$ to 100. As the f_{NL} that we consider falls off rapidly as one goes below the scale $k^{-1} \sim x_{\text{ls}}/60$, a new analysis of the CMB data will be needed to see if it is allowed. Ideally such an analysis should use the full $f_{\text{NL}}(k_1, k_2, k_3)$ provided by a specific curvaton-type model, but for an estimate it may be good enough to identify $f_{\text{NL}}(k)$ with a suitable average over the k_i .

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